A Probabilistic Approach to Trajectory Prediction for Tactical Air Traffic Management

Jordi Vilà–Valls

Centre Tecnològic de Telecomunicacions de Catalunya
Statistical Inference for Communications and Positioning Department

Workshop Tematic Challenge #2
Data-driven Trajectory Prediction

6 November 2018

Intro

- Tactical Trajectory Prediction $\rightarrow$ Estimation/Detection problem
- State-space formulation $\rightarrow$ aircraft dynamics (point mass model), unknown wind forces, on-ground/on-board measurements, known inputs (e.g., engine thrust, flight path angle, bank angle)

The discrete state-space model is defined as

$$\begin{cases} \mathbf{x}_k &= f_k(\mathbf{x}_{k-1}, \mathbf{v}_k, \mathbf{u}_{k-1}) \\ \mathbf{y}_k &= h_k(\mathbf{x}_k, \mathbf{n}_k, \mathbf{u}_k) \end{cases}, \text{ or } \begin{cases} \mathbf{x}_k &\sim p_v(\mathbf{x}_k|\mathbf{x}_{k-1}, \mathbf{u}_{k-1}) \\ \mathbf{y}_k &\sim p_n(\mathbf{y}_k|\mathbf{x}_k, \mathbf{u}_k) \end{cases}$$

Filtering: estimate states using past and current observations: $p(\mathbf{x}_k|\mathbf{y}_{1:k})$ with $\mathbf{y}_{1:k} \triangleq (\mathbf{y}_1, \ldots, \mathbf{y}_k)^T$.

Prediction: estimate states using past observations: $p(\mathbf{x}_{k+\ell}|\mathbf{y}_{1:k})$ with $\ell \geq 1$.

Why such interest in the posterior? The posterior distribution allows us to compute estimators, addressing optimality in many senses,

$$\hat{\mathbf{x}}_t^{\mathrm{MMSE}} = \mathbb{E}\{\mathbf{x}_t|\mathbf{y}_{1:t}\} = \int \mathbf{x}_t p(\mathbf{x}_t|\mathbf{y}_{1:t}) d\mathbf{x}_t \text{ or } \hat{\mathbf{x}}_t^{\mathrm{MAP}} = \arg\max_{\mathbf{x}_t} \{p(\mathbf{x}_t|\mathbf{y}_{1:t})\}$$

Bayesian Filtering

- We are interested in the marginal (filtering) distribution $p(\mathbf{x}_t|\mathbf{y}_{1:t})$. This density can be solved sequentially

$$\cdots \longrightarrow p(\mathbf{x}_{t-1}|\mathbf{y}_{1:t-1}) \longrightarrow \underbrace{p(\mathbf{x}_t|\mathbf{y}_{1:t-1})}_{\text{prediction}} \longrightarrow \underbrace{p(\mathbf{x}_t|\mathbf{y}_{1:t})}_{\text{update}} \longrightarrow \cdots$$

Prediction step: $p(\mathbf{x}_t|\mathbf{y}_{1:t-1}) = \int p(\mathbf{x}_t|\mathbf{x}_{t-1})p(\mathbf{x}_{t-1}|\mathbf{y}_{1:t-1})d\mathbf{x}_{t-1}$

Update step: $p(\mathbf{x}_t|\mathbf{y}_{1:t}) = \frac{p(\mathbf{y}_t|\mathbf{x}_t)p(\mathbf{x}_t|\mathbf{y}_{1:t-1})}{\int p(\mathbf{y}_t|\mathbf{x}_t)p(\mathbf{x}_t|\mathbf{y}_{1:t-1})d\mathbf{x}_t}$

- Standard Filtering Solutions:
  - Linear/Gaussian SSM → Kalman filter (KF)
  - Nonlinear/Gaussian SSM → Extended Kalman filter (EKF) & Sigma-point Kalman Filters (based on deterministic sampling)
  - Nonlinear/Non-Gaussian SSM → Gaussian Sum Filters & Sequential Monte Carlo Methods (Particle Filters)

Dynamic Complex System: nonlinear, non-Gaussian, potentially high-dimensional, multi-object, and with a certain model mismatch.

Standard Filtering Techniques: KF, EKF, SPGF, GSF, SMC
ALL these methods REQUIRE a perfect knowledge of the system and FAIL in non-Gaussian high-dimensional complex systems

Robust statistical inference in general Dynamic Complex Systems?

i) How can we obtain an optimal estimation/prediction in such complex dynamic system (with unknown system parameters)?

ii) How can we deal with deviations from model assumptions (uncertainties, inaccuracies, model mismatch)?

iii) Because of the safety-critical nature of the problem, which is the trade-off between optimality and robustness?

iv) Do the methodologies scale properly with the number of aircrafts present in the airspace of interest (multi-aircraft TP)?

v) Which are the optimal detection metrics to avoid heuristic rules (e.g., conflict detection and resolution)?

Advanced SSP for Next Generation TP

Unknown Noise Statistics (density/parameters)?

- Known Gaussian distribution → Bayesian covariance estimation
- Known unimodal non-Gaussian distribution → hierarchically Gaussian models to infer its parameters within the filter
- Unknown noise densities → Bayesian nonparametric techiques

Multiple Flight Dynamic Models?

- Interacting Multiple Model (IMM) filters
- System model identification techniques

Unknown System Parameters?

- Nested particle filtering architectures using hierarchical models
- Particle Markov Chain Monte Carlo (pMCMC) strategies

CTTC

Centralized or Cooperative Processing?

- On-ground ATC centralized signal processing
- On-board distributed signal processing (e.g., self-separation)

Robust Time-varying Multiple Aircraft TP?

The most general case: unknown and time-varying number of aircrafts in the air space, robustness to unknown system parameters, considering multiple dynamic models, within a distributed architecture.

To summarize:

Different advanced SSP techniques may be relevant to TP for the design of next generation ATM systems, and most of them have not been applied to such context

An interdisciplinary ATM/SSP approach may lead to significant contributions in the field.