The Bridge Between Optimization and Simulation: Application to APOC

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OPTIMIZATION

Optimization

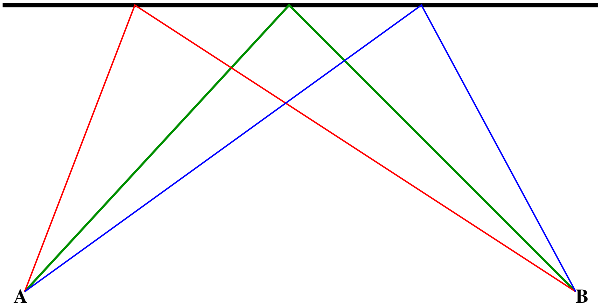
$$\min_{\vec{X}} y = f(\vec{X}) \text{ or } \max_{\vec{X}} y = f(\vec{X})$$

where

- $\vec{X}$ is the vector of decision variables subject to some constraints
- $y = f(\vec{X})$ is the objective function

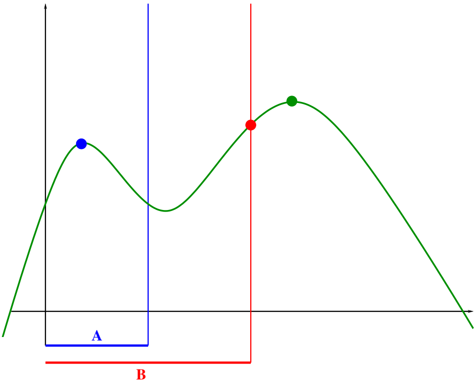
Optimization

- Heron from Alexandria (10-AD) introduced one of the first optimization problem : *Optical shortest path*



Constraints

Unconstrained optimization versus constrained optimization



Discrete or Continuous Space

- **Continuous**

$$\mathcal{X} = \mathcal{U}_1 \times \mathcal{U}_2 \times ... \times \mathcal{U}_m$$
$$\mathcal{I}_i \subset \mathbb{R}$$

- **Discrete**

$$\mathcal{X} = \mathcal{I}_1 \times \mathcal{I}_2 \times ... \times \mathcal{I}_n$$
$$\mathcal{I}_i \subset \mathbb{Z}$$

- **Mixed**

$$\mathcal{X} = \mathcal{U}_1 \times \mathcal{U}_2 \times ... \times \mathcal{U}_m \times \mathcal{I}_1 \times \mathcal{I}_2 \times ... \times \mathcal{I}_n$$

Example of Continuous Problem

FIGURE – Several hundred millions produced every day...

What has to be optimized ?

For a given targeted volume, one want to find the one with the minimum surface

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Mathematical Model

$$\begin{cases} \min Surf = 2.\pi.r^2 + 2.\pi.r.h \\ sc \\ \pi.r^2.h = V_t = 125\text{cm}^3 \end{cases}$$

Example of Discret Problem

What has to be optimized ?

One want to maximize the value of the objects we can put in the bag.

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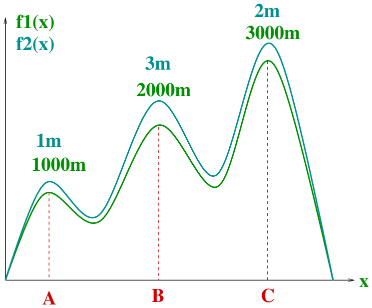
Mathematical Model

$$\vec{x} = (x_1, x_2, ..., x_N) = (0, 1, ..., 1)$$

$$\begin{cases} \max \sum_{i=1}^{i=N} x_i . v_i \\ sc \\ \sum_{i=1}^{i=N} x_i . p_i \leq P_{max} \end{cases}$$

Mono or Multi-objective

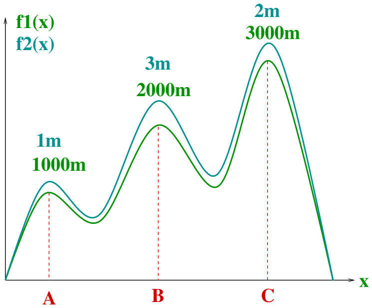
$y = f(\vec{x})$ or $\vec{y} = f(\vec{x})$



- $B >> A$ $C >> A$

Mono or Multi-objective

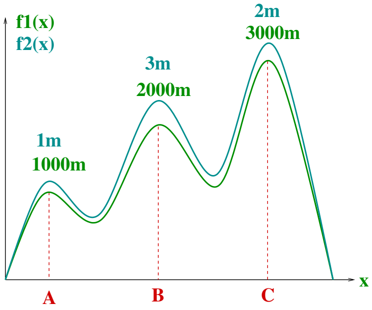
$y = f(\vec{x})$ or $\vec{y} = f(\vec{x})$



- $B >> A$ $C >> A$
- How can we compare B and C ?

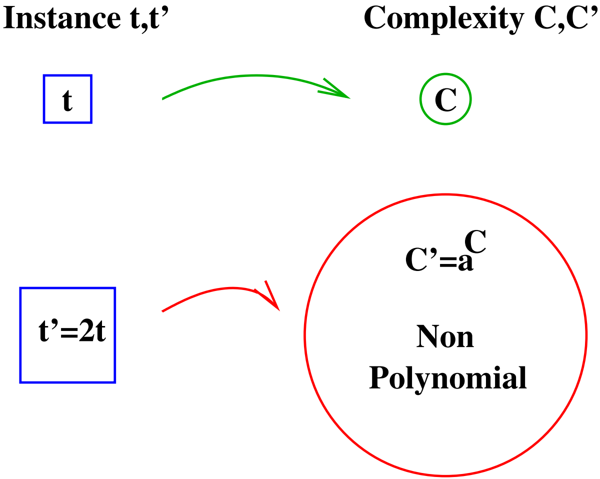
Mono or Multi-objective

$y = f(\vec{x})$ or $\vec{y} = f(\vec{x})$



- $B >> A$ $C >> A$
- How can we compare B and C ?
- No more full ranking between solutions...

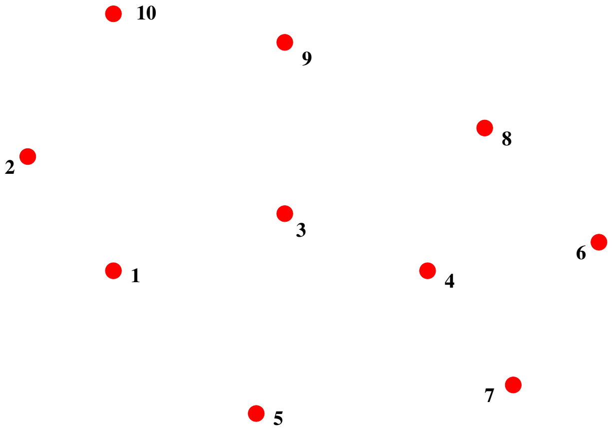
Complexity



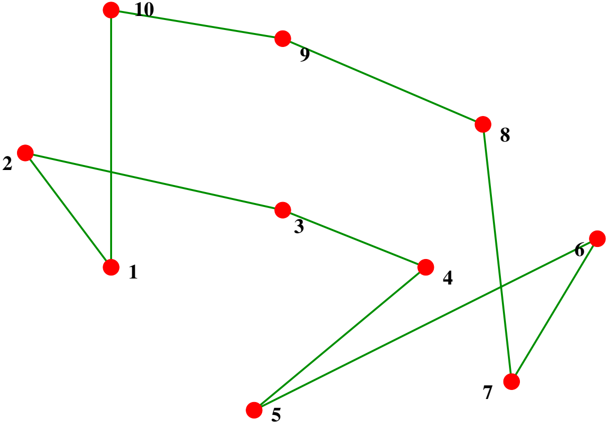
Complexity

Complexity	Description
O(1)	constant
O(log(n))	logarithmic
O(n)	linear
O(n.log(n))	log-linear
O(n^2)	quadratic
O(n^k, $k > 1$)	polynomial
O(2^n)	exponential
O(n !)	factorial

Example of Highly Combinatoric Problem

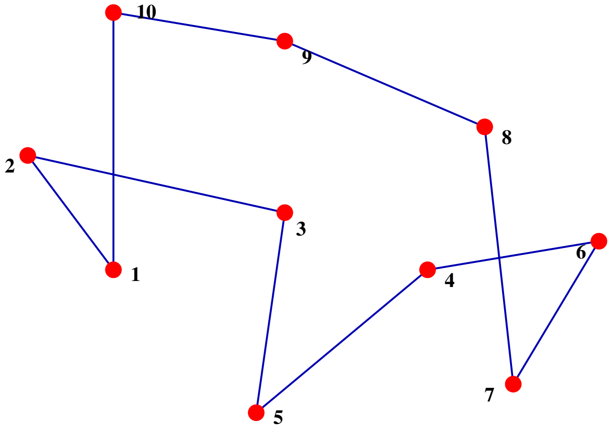


Example of Highly Combinatoric Problem



1	2	3	4	5	6	7	8	9	10

Example of Highly Combinatoric Problem



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What has to be optimized ?

One want to minimize the length of the tour (hamiltonian cycle).

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Mathematical Model

$$\min d(\vec{x}) = \sum_{i=1}^{i=n-1} d(x_{i+1} - x_i) + d(x_n - x_1)$$

Number of tours : N !

Complexity

n	O(1)	O(log(n))	O(n)	O(n.log(n))	O(n^2)	O(2^n)	O(n !)
10	1	1	10	10	100	$1.024\ 10^{3}$	$3.628\ 10^{6}$
20	1	1.30	20	26.02	400	$1.048\ 10^{6}$	$2.432\ 10^{18}$
30	1	1.47	30	44.31	900	$1.073\ 10^{9}$	$2.652\ 10^{32}$
40	1	1.60	40	64.08	1600	$1.099\ 10^{12}$	$8.159\ 10^{47}$
50	1	1.69	50	84.94	2500	$1.125\ 10^{15}$	$3.041\ 10^{64}$
60	1	1.78	60	106.68	3600	$1.152\ 10^{18}$	$8.320\ 10^{81}$
70	1	1.84	70	129.15	4900	$1.180\ 10^{21}$	$1.197\ 10^{100}$
80	1	1.90	80	152.24	6400	$1.208\ 10^{24}$	$7.156\ 10^{118}$
90	1	1.92	90	175.88	8100	$1.237\ 10^{27}$	$1.485\ 10^{138}$
100	1	2.00	100	200.00	10000	$1.267\ 10^{30}$	$9.332\ 10^{157}$

Comparison of $O(2^n)$ and $O(n!)$

1 nano-second for one computation

n	$O(2^n)$	$O(n!)$	$\frac{n!}{2^n}$
10	1 micro second	3.6 mili seconds	$3.6\ 10^{3}$
20	1 mili second	77 years	$2.3\ 10^{12}$
30	1 second	$8.4\ 10^{15}$ years	$2.47\ 10^{23}$
40	18 minutes	$2.5\ 10^{31}$ years	$7.4\ 10^{35}$
50	13 days	$9.6\ 10^{47}$ years	$2.7\ 10^{49}$
60	36 years	$2.6\ 10^{47}$ years	$7.2\ 10^{63}$
70	$37\ 10^{3}$ years	$3.8\ 10^{83}$ years	$1\ 10^{79}$
80	$38\ 10^{6}$ years	$2.2\ 10^{102}$ years	$5.9\ 10^{94}$
90	$39\ 10^{9}$ years	$4.7\ 10^{121}$ years	$1.2\ 10^{111}$
100	$40\ 10^{12}$ years	$2.9\ 10^{141}$ years	$7.3\ 10^{127}$

Methodology

Questions

- What is our objective ?
- What are the parameters on which we can (should) act ?
- What are the constraints ?

Review of properties of the objective function, of the state space and constraints

- What is the right level of abstraction of the mathematical model ?
- What is the dimension of the state space ?
- What is the maximum time we have, to perform the computation ?
- What is the memory size of a point of the state space ?
- Can we find similarity with some other problems ?

Optimization Methodology

Closed Forms

Fermat's theorem, Necessary conditions

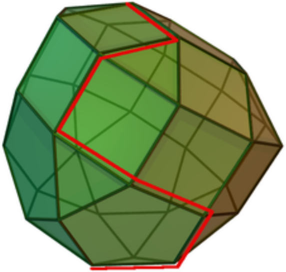
$$\nabla f(\vec{x}^*) = 0$$

$$\nabla^2 f(\vec{x}^*) \text{ is positive semi-definite.}$$

Deterministic Approaches

Linear Programming

$$min\ f(\vec{x}) = A^T.\vec{x} \text{ subject to } D.\vec{x} \leq \vec{b}, \vec{x} \geq 0$$



Simplex Algorithm (Dantzig 1963), Interior Point Algorithm (John Von Neuman-1948, Karmarkar-1984)

Order Zero Nelder Mead 1965, Powell 1973

Nelder Mead Video!

Order one : Descent Algorithm

Descent Algorithm Video !

Order two : Trust Region Algorithms

Examples : Newton, BFGS (Local Quadratic Model of the criterion)

Global Deterministic Approaches

Enumeration

- When the principle of locality is not satisfied (pics on a flat landscape)

Enumeration

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- GPU help for such enumeration.

Branch and Bound

- First proposed by A. H. Land and A. G. Doig in 1960.

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- **Key ! A bound of the objective function is needed**
- Often used for Integer Programming (with LP relaxation)

Stochastic Approaches

Random moves in the state space

Simulated Annealing

Emulation of the physical process whereby a solid is slowly cooled

Simulated Annealing Video !

(Kirkpatrick, Gelett and Vecchi 1983, Cerny 1985)

Application to TSP

Simulated Annealing for TSP!

EA uses mechanisms inspired by biological evolution, such as reproduction, mutation, recombination, and selection

Artificial Evolution Video!

Holland 1975, Goldberg 1989,L.J.Fogel 1966,Schwefel and Rechenberg 1965,Koza 1992

Classification

Classification

Deterministic

- Local
  Non linear programming (NLP : order 0,1 or 2),...
- Global Linear programming (LP), Branch and Bound, Homotopy
  Continuous Deformation Methods (Homotopy),...

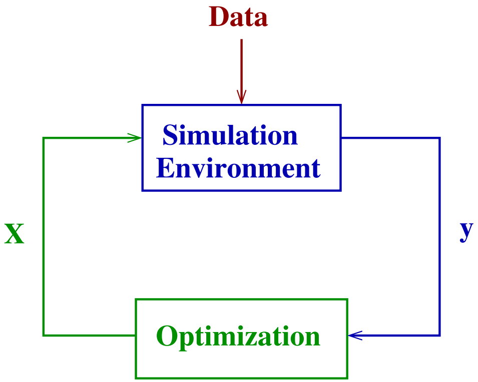
Stochastic

- Global
  Enumeration, Branch and Probability Bound , Taboo Search,
  Simulated Annealing, Artificial Evolution,...

	Lin	N Lin	Cont	Disc	L Di	GLO	Mu M	Mu O
LP	●		●					
NLP	•	●	●					
B&B	•	●	•	●		•		
TAB	•	•	•	•	●	●		
SA	•	•	•	•	●	●		
BPB	•	•	•	•	●	●		
HOM	•	•	•	•	●	●		
AE	•	•	•	•	●	●	●	●

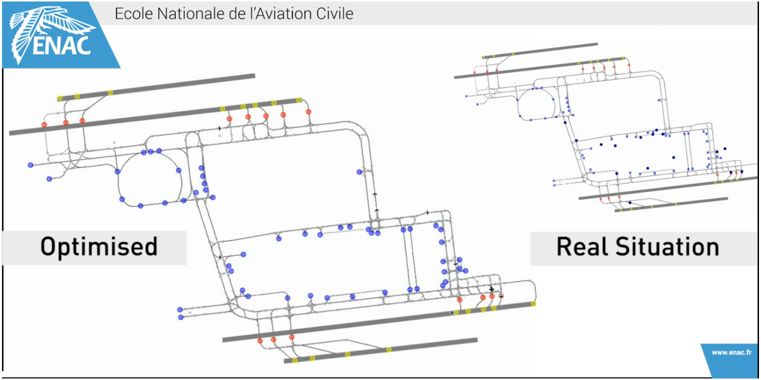
Simulation Based Objective Function Evaluation

- Thanks to the new CPU/GPU facilities one can easlily plug a simulation into an optimization algorithm (depending on the available compution time)



Paris CDG Taxi Optimization

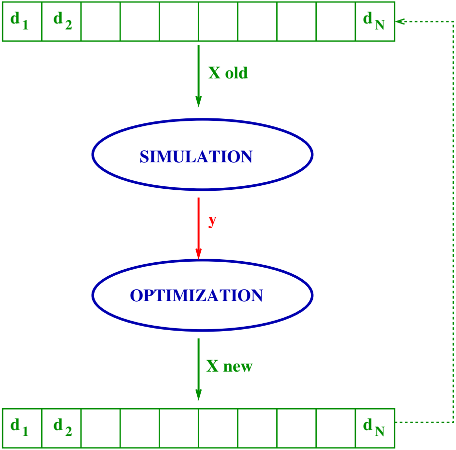
Simulation based evaluation coupled with a sliding window principle



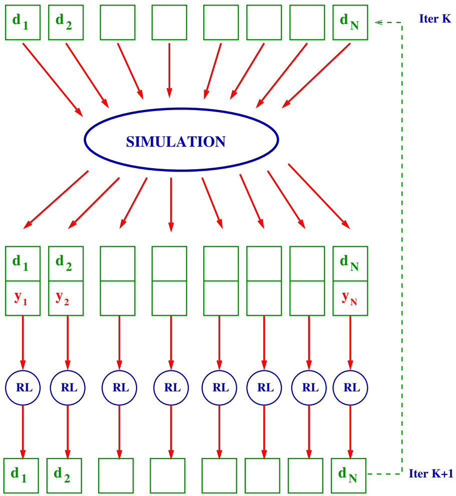
Paris CDG Taxi Optimization (video)!

Centralized versus Decentralized Optimization

Centralized Optimization



Global optimum but request a full communication and coordination between decision makers (equity issues)



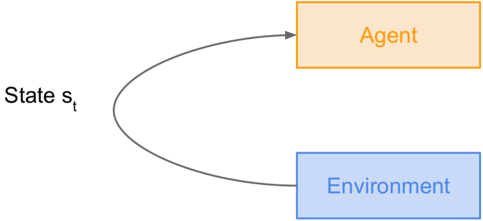
Less optimal but only local knowledge is requested in order to build near optimal strategy ($\Rightarrow$ Emergence).

Reinforcement Learning

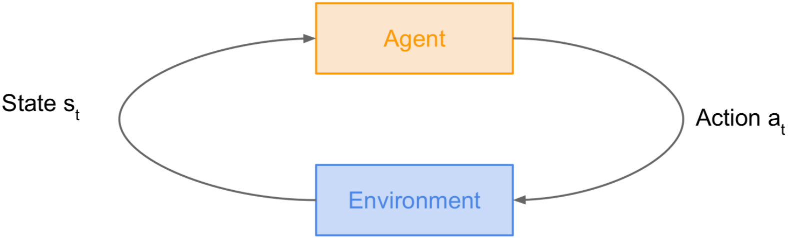
- Problems involving an agent interacting with an environment, which provides numeric reward signals
- Learn how to take actions in order to maximize reward

Reinforcement Learning

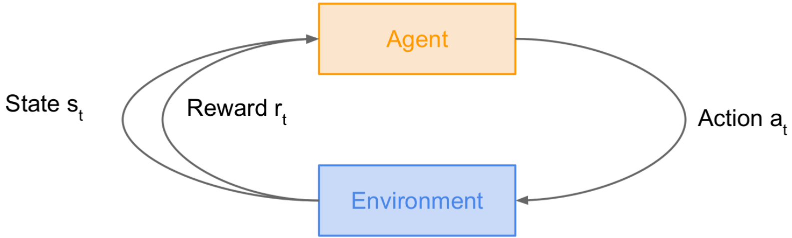
Reinforcement Learning



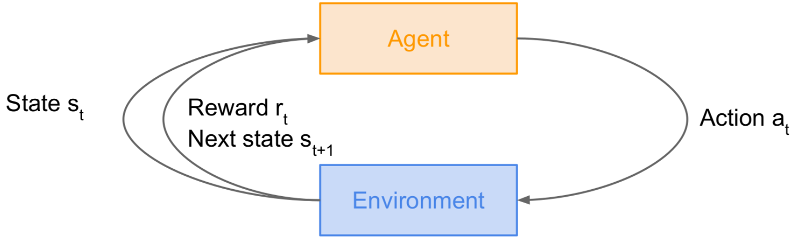
Reinforcement Learning



Reinforcement Learning



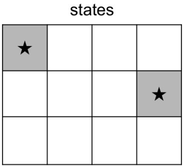
Reinforcement Learning



Markov Decision Process

A simple MDP: Grid World

actions = {
1. right
2. left
3. up
4. down
}

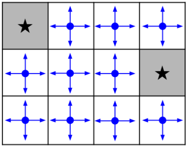


Set a negative "reward" for each transition (e.g. r = -1)

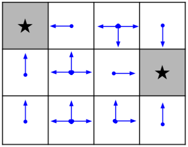
Objective: reach one of terminal states (greyed out) in least number of actions

Markov Decision Process

A simple MDP: Grid World



Random Policy



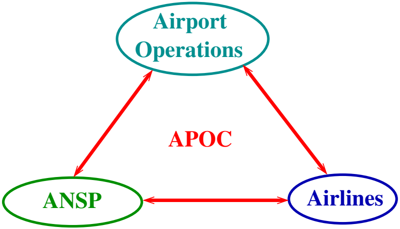
Optimal Policy

Reinforcement Learning Example (2)

Game Expertise Learning !

Application to APOC

APOC Structure



- Use Machine Learning (RL, ...) approach to establish the best collaboration policies which enhance exchanges between agents in order to maximize the overall performances of the airport operations.
- Determine by ML the best incentives to target this goal.

Work proposed in the PhD

- Development intelligent agents that simulate the different stakeholders present in an APOC environment (airlines, airport ops and ANSPs).
- Develop a methodology that integrates the causal behaviour of actors in the agent-based environment
- Simulate the operations performed in an APOC making use of the developed agents
- Analyse the behaviour of agents using the virtual environment and propose policies or methods for enhancing cooperation amongst the stakeholders
- Validate the developed approach in a virtual environment

Questions ?